Solutions to Problem 1.

a. Looking at the transition probability diagram, we can see that $\{1, 2\}$ and $\{3, 5\}$ form self-contained Markov chains, and no proper subsets of $\{1, 2\}$ or $\{3, 5\}$ form a self-contained Markov chain.

b. Recurrent states: 1, 2, 3, 5 (these are states that are part of an irreducible set, by part a)
Transient states: 4, 6 (these are states not part of an irreducible set)

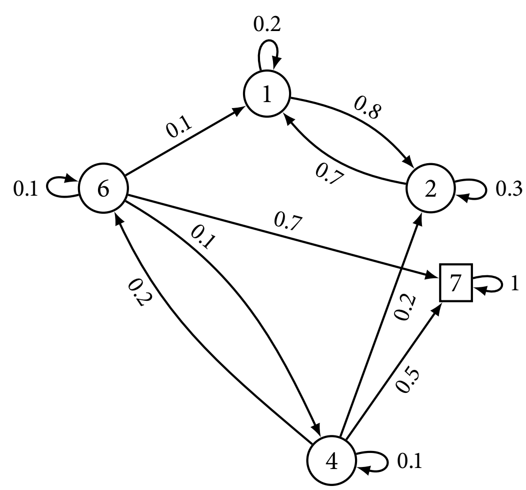
c. Let $\mathcal{R} = \{1, 2\}$. We want π_1. From the transition probability diagram, $\mathbf{P}_{\mathcal{R}\mathcal{R}} = \begin{bmatrix} 0.2 & 0.8 \\ 0.7 & 0.3 \end{bmatrix}$. Therefore,

$$\begin{aligned} \pi_{\mathcal{R}}^{\top} \mathbf{P}_{\mathcal{R}\mathcal{R}} &= \pi^{\top} \\ \pi_{\mathcal{R}}^{\top} &= \mathbf{1} \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} 0.2\pi_1 + 0.7\pi_2 &= \pi_1 \\ 0.8\pi_1 + 0.3\pi_2 &= \pi_2 \\ \pi_1 + \pi_2 &= 1 \end{aligned} \quad \Rightarrow \quad \pi_1 = \frac{7}{15}, \ \pi_2 = \frac{8}{15}$$

So, the long-run fraction of time the UAV spends in region 1 is 7/15.

d. This is a little tricky: the definition of an absorbing probability requires an absorbing state, that is, an irreducible set of states with only one state.

Let's replace states 3 and 5 with a "super state" called 7. We end up with the following transition probability diagram:



Now, let $\mathcal{T} = \{4, 6\}$ and $\mathcal{R} = \{7\}$. We want α_{47}:

$$\alpha_{\mathcal{T}\mathcal{R}} = (\mathbf{I} - \mathbf{P}_{\mathcal{T}\mathcal{T}})^{-1} \mathcal{P}_{\mathcal{T}\mathcal{R}} = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix} \approx \begin{bmatrix} 0.747 \\ 0.861 \end{bmatrix}$$

Therefore, $\alpha_{47} \approx 0.747$.